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Atlanta Fiber System Experiment:

Demountable Single-Fiber Optic Connectors and Their Measurement on Location

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The transfer-molded, single-optical-fiber connector and the fiberguide distribution system implemented for the Atlanta Fiber System Experiment are described. A new technique for measuring the connector and connectorized fiber cable loss in the field was implemented and the results are reported. More recent results of the evolutionary versions of the connector are also reported.

I. INTRODUCTION

The Atlanta Fiber System Experiment was a first test of fiberguide equipment to explore the feasibility of using optical fibers for digital metropolitan trunk transmission systems. It was realized at the onset that high-density systems using cables containing up to 144 fibers[1] would require a fiber interconnection system within the central offices to allow convenient access to all fibers and to permit connecting any number of fibers end to end and to the electronics. A fiberguide interconnection system was therefore implemented for the Atlanta Experiment. This paper describes the interconnection system and the single-fiber connector on which it is based.

II. THE FIBERGUIDE INTERCONNECTION SYSTEM

Figure 1 shows the schematic of the fiberguide interconnection system. The equipment bay containing the fiber optic transmitter and receiver plug-ins and the bay containing the fiberguide distributing frame stand side-by-side and are optically connected by "bay jumpers." The transmitter and receiver circuit boards have fiberguide plugs mounted at the

* P. K. Runge is responsible for the design of the single-optical-fiber connector and S. S. Cheng for their measurement on location.

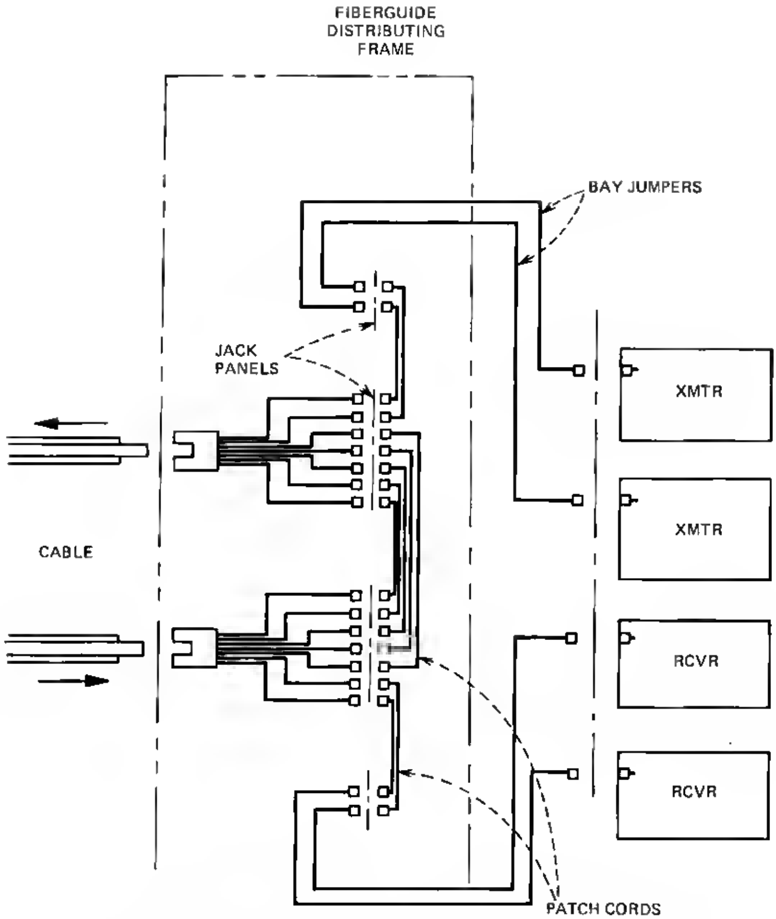


Fig. 1 — Schematic of the fiberguide distribution system.

back so that optical connections to the bay jumpers are made simultaneously with electricrical connections upon insertion. The bay jumpers terminate on a jack panel in the distributing frame. Adjacent to that panel are two jack fields that provide access to both ends of all 144 fibers in the fiberguide cable.

Fiberguide patch cords, typically ½ to 1 meter long, are plugged into the jack fields to provide cable end-to-end connections as shown in Fig. 2. Patch cords also provide the optical link from the equipment jack panel to the cable panels. Optical reconfigurations are all done by moving patch cords at the distributing frame.

The jacks of the distributing frame are connected to cable splice connectors[2] through the individual fibers of the "fan-out" unit. Figure 3 shows a photograph of one fan-out unit taken from the back of the distributing frame at the time the cable splice connection was made. (The

Fig. 2 — The fiberguide distribution frame.

fiberguide cable rises at the left edges of the fan-out units.) The stack of 12 fiberguide ribbons fans out into individual ribbons containing 12 fibers each, each ribbon leading into one organizer tube. Inside the tube, each ribbon fans out into its 12 individual fibers addressing the group of jacks at the opposite end of the tubes. Two of these fan-out units in Fig. 3 were stacked in the back of the distributing frame.

III. SINGLE FIBER CONNECTOR

The heart of the distribution system is the single fiber connector. The connector butt joins two single optical fibers having nominally 110-μm outside diameter and a 55-μm-diameter graded-index core. The mechanical tolerances that have to be maintained in each connection become apparent in Fig. 4. The measured transmission loss is plotted versus axial and lateral misalignment error between two fiber ends with the same core parameters. The loss data were taken with light-emitting diode

Fig. 3 — One fiberguide fan-out box for 144 fibers.

excitation and glycerin as index-matching media in the gap of the joint. The lateral misalignment error contributes mostly to this extrinsic loss of a fiber connection, and for this fiber the sensitivity ratio of small lateral and axial errors is about 20:1.

When fibers are connected at random, variations in their parameters, as diameter, numerical aperture, and profile of the distribution of index of refraction, lead to an additional, intrinsic loss in the connection. Assuming that a connector with 1-dB average transmission loss has 0.5-dB intrinsic loss leaves only 0.5 dB extrinsic loss for the connector design itself. According to Fig. 4, the total lateral offset error then has to be held below about 5 μm. For the development of a demountable connection, that constitutes a formidable task.

Besides the unprecedented tolerance requirements, the total cost of the single-fiber connector will also be a decisive parameter, since a large number of connectors will be needed, and ultimately optical fiber systems have to compete economically against copper facilities. In addition, since, for the Atlanta Experiment itself plus the assorted test equipments, over 1000 plugs (a plug is the male part of the connector containing the optical fiber) were needed, a mass producible connector was desired.

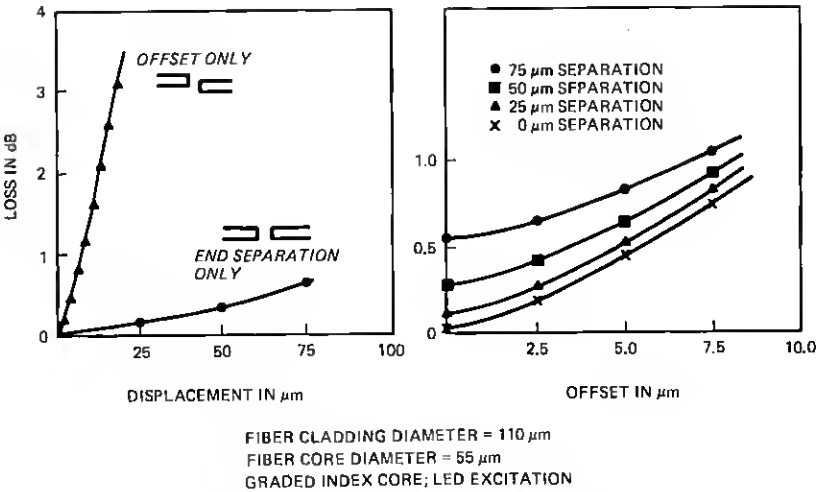


Fig. 4 —Transmission of a single fiber butt joint.

A precision transfer-molding process was developed to produce the plugs shown in Fig. 5. The connector alignment is provided by concentric conical surfaces at the tips of the plugs and by a biconic socket. The molded plug bodies have grooves for additional outside hardware to provide an axial force to seat the connector. The cutout in Fig. 5 is an exploded view of the two optical fiber ends embedded in flexible index-matching cushions.

We have selected the transfer molding process as a mass production process for thermosetting materials that is most gentle on the optical

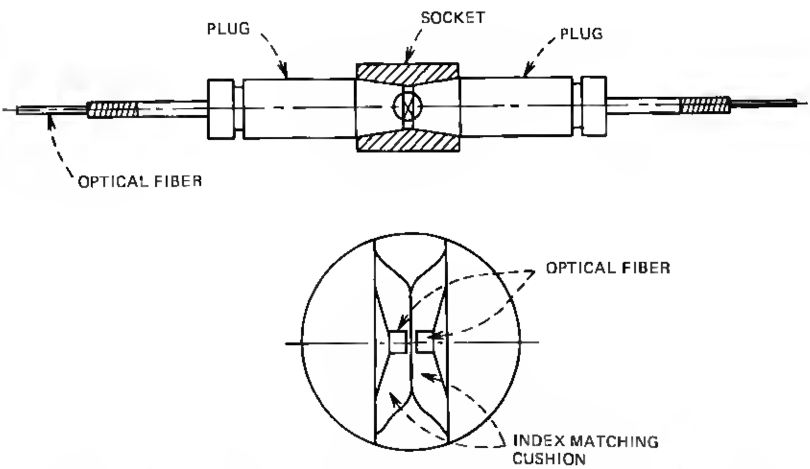


Fig. 5 — Cross section of the single fiber connector detail shows elastomeric index matching cushions.

fiber and guarantees that the mechanical impact of the molding material rarely causes fiber breakage and minimizes deflection. The key part of the mold is a precision die that is cylindrically symmetric without parting lines. It possesses a centering guide hole of nominally 115 μm in diameter, concentric to better than a fraction of a micron with respect to the axis of the tapered surface that constitutes the aligning surface of the molded plug. The diameter of the guide hole equals the nominal fiber outside diameter plus three standard deviations.

A taper was chosen as the aligning surface for a number of reasons.

(*i*) It accommodates molding materials with a varying rate of shrinkage, as long as the material shrinkage is isotropic. Variations from molding to molding do not manifest themselves in lateral misalignments, but rather in the less critical axial error.
(*ii*) It allows easy insertion for low abrasion and good performance reproducibility.
(*iii*) The geometry allows fibers of different diameters to be interfaced.

The molding material is a heavily silica-filled epoxy, which yields plugs of excellent mechanical integrity, extreme toughness, and good surface qualities, including excellent abrasion resistance. After molding, about 2 cm of unprotected fiber protrudes beyond the epoxy body. The angular misalignment between the optical fiber and the taper axis was measured for a large number of plugs by inserting the plug into a brass socket with complementary taper, rotating it, and measuring the walkout of the exposed fiber. The results for the first 397 plugs are indicated in Fig. 6. Ninety percent of all plugs have an angular misalignment of less than 0.8 degree, which is well within the design limit of 1 degree. If desired, a reduction of this angle is possible by improving the guidance of the bare optical fiber in the mold.

We cut the fiber end by inserting the plug into a reference socket and by scoring the fiber at a fixed position with respect to the socket such that two plugs mating in a double conical socket would consistently have a 30-μm gap between fiber ends. The cutting technique of Gloge et al.[3] was employed, where the fiber is simultaneously bent and stressed and then scored.

The most critical tolerance of the fiber optic connector is the eccentricity of the fiber with respect to the axis of the tapered aligning surface. To measure the eccentricity, we used a Ge-doped graded-index fiber that possessed a small depression of the index of refraction at the center of the core. This depression shows up as a dark spot at the center of the illuminated core, about 1 μm in diameter, and allowed us to measure the plug eccentricity to within 0.5 μm.

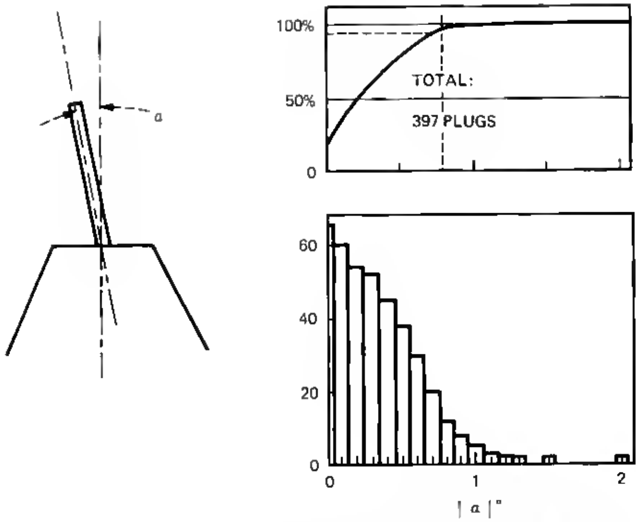


Fig. 6 — Angular error of the first 397 molded plugs.

For this measurement, the plug is inserted into a single steel socket mounted rigidly under a microscope with 400× magnification. By rotating the plugs in this socket, the eccentricities of the optical fibers with respect to the taper axis can be determined. As an example, Fig. 7 shows the location of the centers of 24 plugs molded in die 5 as obtained by this method. Within the accuracy of the measurements, the samples are scattered at random and do not reveal a systematic error. Figure 8 is a plot of the results of 144 measurements of the absolute value of the eccentricity (the angular information is lost). The average error is about 1.2 μm, and 90 percent of the measured plugs had an error of less than 2 μm. A few plugs had eccentricities in excess of 4 μm, but these could be traced to moldings imperfections as, for example, entrapped air bubbles on the critical taper surfaces or a deformation of the taper itself on demolding.

The measured eccentricity, of course, includes the eccentricity of the fiber itself. Although it would have been desirable to obtain statistical information about the eccentricity of the fiber, these data are very difficult to measure to an accuracy of a fraction of a micron. A few sample measurements indicate an average fiber eccentricity of 0.75 and a maximum value of 1.2 μm. Eccentricities in the centering mechanism account for an additional error of the order of 0.5 to 1 μm. The average eccentricity of about 1.2 μm in Fig. 8 thus can easily be explained in terms of those two factors. In view of these facts, we have to conclude that the eccentricity of the plug resulting from nonlinear shrinkage of the epoxy is not dominant and must, on an average, be smaller than about 1 μm.

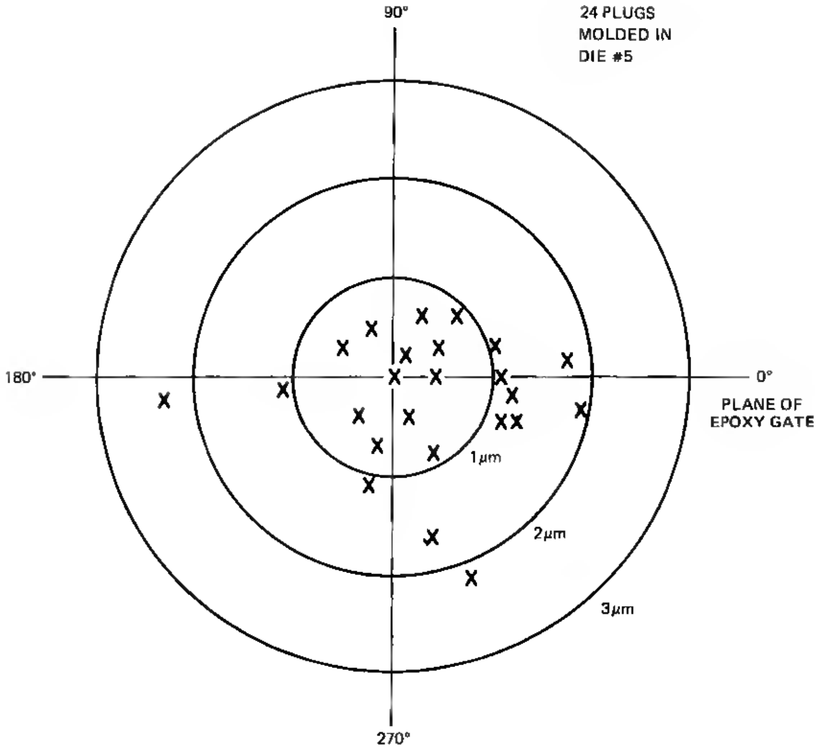


Fig. 7 — Distribution of core center location for first 24 plugs molded in one die.

The process for molding the precision biconic socket was not developed in parallel witb the process for plug molding, so that at the time of tbe Atlanta Experiment only metallic sockets were available. Figure 9 is a photograph of plugs molded on nylon coated fibers mated in a machined brass biconic socket.

The socket in Fig. 9 has an observation hole through which the shadowgraphs of Figs. 10 and 11 were taken. Figure 9 shows the two fiber ends opposing each other with an air gap of about 41 μm. To reduce the transmission loss from Fresnel reflection and refraction and also to protect the fiber ends from contamination, transparent cushions of silicone rubber were applied to each plug. Figures 11a through 11c are three steps in a sequence showing two plugs being pulled apart. As Fig. 11c demonstrates, an index-matching medium witbout interface is formed between the fiber ends. This medium reduces tbe transmission loss by about 0.4 dB when compared to the loss in the air gap of Fig. 10. (Figures 10 and 11 have the same magnification.)

Two types of metallic biconic sockets were available for tbe experiment: sockets made by hydroforming copper sleeves onto precision steel mandrels and sockets machined from solid brass. The hydroformed

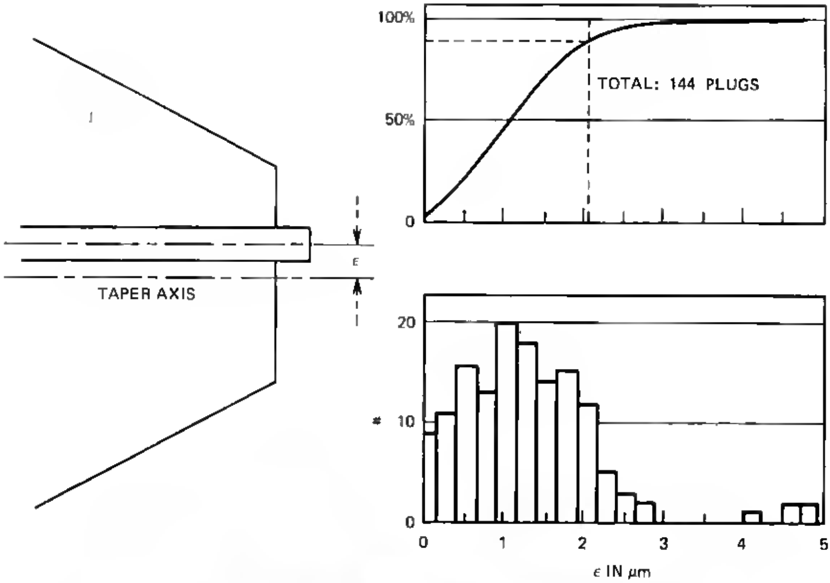
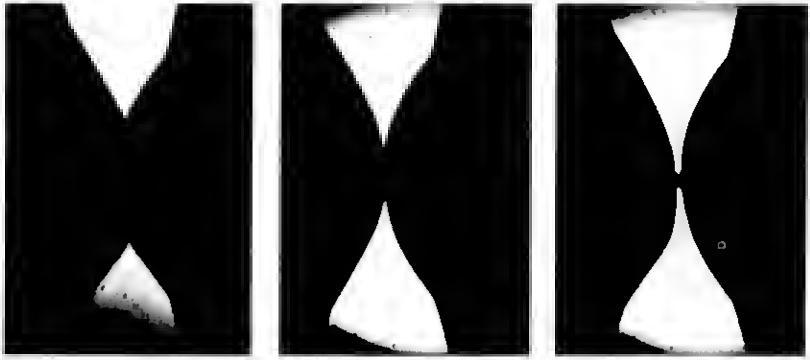


Fig. 8 — Distributions of eccentricity errors of the first 144 molded plugs.


Fig. 9 —Two plugs mated in a brass biconic socket. Note the observation hole.


Fig. 10—Shadowgraph of opposing fiber ends of two plugs mated in the socket of Fig. 9.

sockets showed eccentricities between 5 to 10 μm and had too much resiliance on insertion of the plugs, so that the gap between fiber ends had to be increased to 100 μm. The machined brass sockets had eccentricities in the range from 0 to 5 μm and were in addition stiffer, which allowed the gap to be reduced to 65 μm.



(a) (b) (c)

Fig. 11—(a) Shadowgraph of two plugs with flexible transparent "buttons" for index match mated in the biconic socket (same magnification as Figure 10). (b) and (c) One connector is gradually unseated; button surfaces cling together.

IV. INTERCONNECTION HARDWARE

The quick connection at the rear of the transmitter and receiver cards is provided by mounting the molded plug in the spring-loaded jig as shown in Fig. 12. Two pins guide the plug into the floating biconic socket mounted on the frame. The plug is connected to the receiver or transmitter package on the circuit board by a "pigtail," which is a nylon-

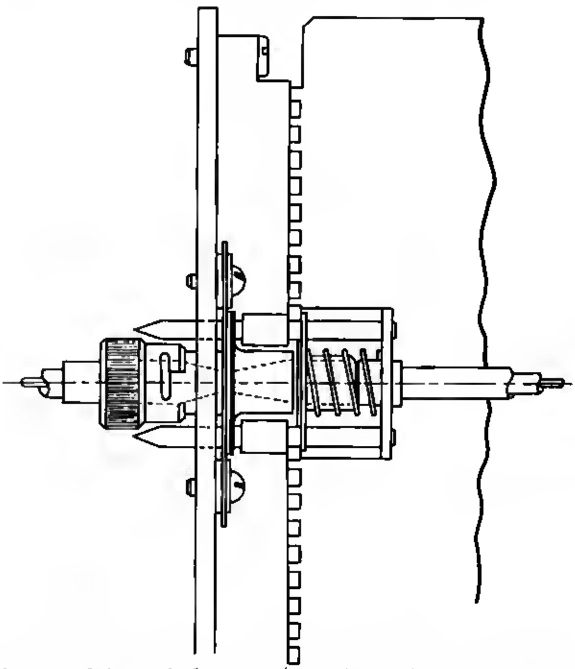


Fig. 12 — Schematic drawing of the fiberguide card connector.

coated fiber in a loose protective sheath that is looped to permit easy motion when the plug engages. Figure 13 shows the three plug-ins that comprise a full repeater photographed from the connector side.

The armor for an optical fiber patch cord must protect the fiber against excessive bending, compression, and elongation forces, yet it should be flexible and light in weight to ease the forces on the optical connectors. The experimental patch cords utilized the standard protection of a miniature coax cable with the optical fiber replacing the center conductor. Its Teflon® inner jacket protects the optical fiber from contact with the metallic braid and offers protection against sharp bends of the cable. The metallic braid protects, within limits of about 20 kg, against elongating forces. The optical connectors were molded directly onto the ends of the patch cord. The cords have not been tested systematically to failure, but they are surprisingly rugged, and to our knowledge there have been no failures in normal usage from a fiber breaking in the sheath.

The fiber connector of Fig. 3 required some outside hardware to exert the seating force and also to protect the connector ends. For convenience, the outer shell of a standard BNC coax connector was modified and served that purpose well. Thus, the patch cords being handled in Fig. 2 only look like coax cables; in reality they are fiberguides.

The fan-out was constructed by molding plugs onto 2-m-long, nylon-coated fibers. These were assembled into ribbons, 12 at a time, leaving about one-half meter of coated fiber between the ribbon and plug. The 12 plugs were mounted on a 12-jack panel, the panel was mounted on the distributing frame, and the ribbon was coiled loosely in the organizer tube behind each panel. The ribbons were then brought together at the back of the distributing frame and mounted into a stack of grooved wafers for connection to the cable.

V. A STATISTICAL LOSS MEASUREMENT METHOD

The loss measurement set (Fig. 14) consists of an optical transmitter with a stable laser as the source, and an avalanche photodiode detector (APD) operating in the linear range as the optical power detector. The optical link may consist of up to three jumpers and two fibers. These fibers are part of the 144 fibers in the 640-m long fiber cable in the Atlanta Experiment.

Figure 14 shows a simplified measurement sequence that statistically determines the loss of three jumpers and two fibers. The first setup is for the baseline calibration; the transmitter is connected to the receiver via a variable optical attenuator and/or reference jumpers and fibers for setting a proper input level. The baseline level is chosen such that the received power level for all subsequent setups will be within the linear operating range of the APD. In each setup, the received power is measured

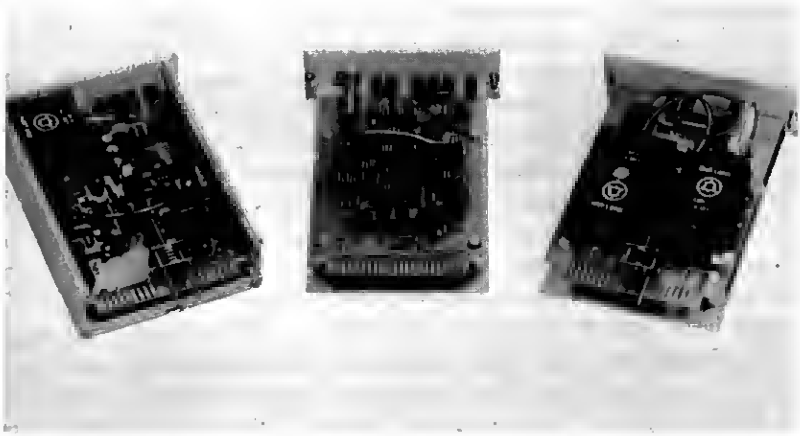
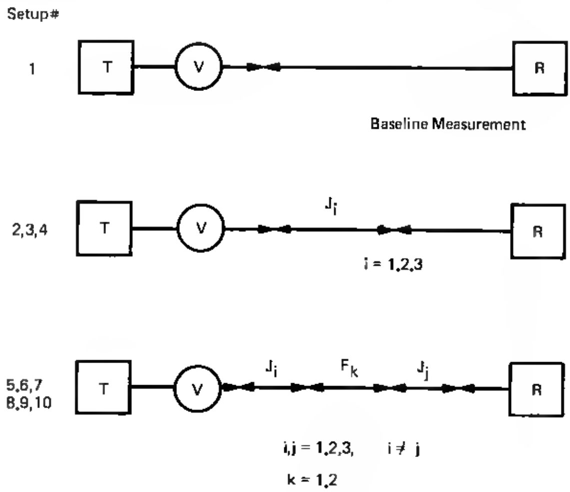


Fig. 13—A complete fiberguide regenerator with: Left: receiver module with fiberguide card connector. Center: decision and retiming module. Right: laser transmitter module with fiberguide card connector.



NOTES:

(1) IN EACH SETUP, AT LEAST 10 POWER MEASUREMENTS ARE MADE TO ENABLE A MEANINGFUL AVERAGING OF MEAN AND RMS.

(2) V REPRESENTS A VARIABLE ATTENUATOR AND/OR SEVERAL REEERENCE JUMPERS AND EIBERS EOR SETTING A PROPER BASELINE LEVEL.

Fig. 14 —A simplified sequence of statistical loss measurement.

many times so that a mean and an rms level can be determined. In each measurement, the connection between the transmitter and the receiver jumpers is disconnected and then reconnected with a slightly different

orientation. The purpose is to enable a meaningful averaging of loss over all possible connector orientations.

In setups 2 through 4, one of the three jumpers is added to the optical link. Again, in each power measurement, the jumper is disconnected on both ends and reconnected at a different angle. Setups 5 through 10 consist of the set and one possible combination of two jumpers and one fiber. Here, in each measurement, both jumpers are disconnected and reconnected each time.

At tbe conclusion of all measurements, a least-square technique is applied to these data to determine the best fits for the six unknowns (five loss elements and the baseline level of the set). The simultaneous determination of many loss elements reduces the inconsistency that may occur in the measurement. The loss is represented by a mean value and an rms value. The rms value is contributed partially by the measurement inaccuracy and partially by the offset and the angular dependence of the connector loss. The mathematical basis for such an approach is presented in the appendix.

The success of this metbod depends on the following assumptions and precautions:

(*i*) The APD should be operated in a linear range to serve as an optical power detector. For the optical receiver, −10 to −45 dBm is a suitable range. The stability of the laser is also important. Overnight monitoring of the laser power indicates a variation of less than 0.1 dB.

(*ii*) The loss of various components in the optical link must add up linearly. To insure this, additional mode mixing is introduced through one variable optical attenuator and/or additional reference jumpers and fibers in front of the loss elements to be measured. Once the attenuator position is set and the reference jumpers and fibers are in place, they remain intact throughout the rest of the measurement sequence. The reference jumper and fiber are chosen on the basis that tbe connection yields a minimum uncertainty in the repeated baseline measurements. With carefully chosen reference jumper and fiber, the uncertainty is typically 0.06 dB.

(*iii*) To take full advantage of the redundant measurements, a jumper-to-fiber mix of 3 to 2 is preferred over 3 to 1 or 4 to 1. The ratios of number of measurements to number of unknowns are 10 to 6, 7 to 5, and 9 to 6 respectively. The increased redundancy reduces the contribution from poor or inconsistent measurements.

An HP 9830A calculator was programmed to perform the power measurement and the data reduction. To double-check the results, we made other measurements that include some previously measured jumpers and fibers. Two such measurements were made at more than a 1-month

interval using different transmitters and receivers. The agreement has been quite good and was sufficient to indicate that these measurements are reliable and repeatable.

VI. MEASURED LOSS OF CONNECTORS AND FIBERS IN THE ATLANTA EXPERIMENT

Tables I and II summarize the measured jumper and fiber loss. The consistency among these measurements is generally good. That is, the difference between the mean loss of two measurements is usually less than the combined uncertainty of the measurements. Out of the 27 elements measured, there are three exceptions to this rule, J72, F1-4, and F3-1. The reason for some of them may be purely statistical, but more likely it may be because of a permanent deterioration of the surface condition at the fiber tip of the connector. This is concluded on the basis that the increase in loss invariably occurs after hundreds of times of handling and that, once the loss is increased, it is irreversible.

The average insertion loss of jumpers using hydroformed sockets ranges from 1.0 to 3.1 dB with an uncertainty between 0.1 to 0.4 dB. The average loss of all such jumpers measured is 1.75 ± 0.26 dB.

Jumpers using machined sockets have a mean loss ranging from 0.5 to 1.9 dB and an uncertainty from 0.1 to 0.4 dB. The average loss of this

Table I — Jumper loss in dB

Jumper No.*	1st	2nd	3rd	Avg.
72	2.4 ± 0.3	3.6 ± 0.4		2.4 ± 0.3†
76	1.6 ± 0.3			1.6 ± 0.3
77	3.1 ± 0.4	2.5 ± 0.2	2.5 ± 0.2	2.7 ± 0.3
80	1.7 ± 0.3			1.7 ± 0.3
82	2.0 ± 0.3			2.0 ± 0.3
84	1.5 ± 0.2			1.5 ± 0.2
87	1.0 ± 0.1			1.0 ± 0.1
91	1.0 ± 0.3	1.1 ± 0.1		1.0 ± 0.2
149	1.6 ± 0.3			1.6 ± 0.3
150	0.9 ± 0.2	0.6 ± 0.2		0.7 ± 0.2
154	1.6 ± 0.1	1.7 ± 0.1		1.6 ± 0.1
157	1.1 ± 0.3			1.1 ± 0.3
160	1.5 ± 0.2	1.7 ± 0.3		1.6 ± 0.2
161	1.2 ± 0.2			1.2 ± 0.2
168	0.7 ± 0.3			0.7 ± 0.3
169	1.8 ± 0.4			1.8 ± 0.4
172	1.4 ± 0.2	1.9 ± 0.4		1.6 ± 0.3
174	1.2 ± 0.2	1.2 ± 0.2	1.3 ± 0.1	1.3 ± 0.2
176	0.5 ± 0.2	0.8 ± 0.1	0.8 ± 0.2	0.7 ± 0.2
180	0.5 ± 0.1	0.7 ± 0.2		0.6 ± 0.1

* Jumpers No. 72 through 91 are with hydroformed socket, No. 149 and up are with machined socket.
† The second measurement is excluded because the jumper by then might have suffered permanent degradation due to repeated use.

Table II — Fiber loss in dB

Fiber No.	1st	2nd	3rd	4th	Avg.
1-4	6.5 ± 0.2	8.0 ± 0.4*			6.5 ± 0.2†
1-11	8.1 ± 0.4*	8.3 ± 0.6	8.1 ± 0.3	8.3 ± 0.3*	8.2 ± 0.4
2-1	4.7 ± 0.3	4.7 ± 0.3			4.7 ± 0.3
2-8	4.9 ± 0.4*	5.6 ± 0.5			5.2 ± 0.5
3-1	5.0 ± 0.3	5.9 ± 0.2			5.0 ± 0.3†
5-2	5.5 ± 0.4	5.2 ± 0.6*			5.4 ± 0.5
7-2	6.6 ± 0.5*	6.8 ± 0.4			6.7 ± 0.4

* Jumpers with hydroformed socket.
† The tips on the fiber connectors might have suffered permanent degradation after repeated use. The last measurement is excluded in obtaining the average loss.

type of jumper is 1.22 ± 0.21 dB, indicating convincingly that this type of socket is superior to the former type. As indicated before, machined sockets have tighter tolerances, thus providing better alignment and less loss. The small improvement in the loss uncertainty (from 0.26 to 0.21 dB) suggests that the measurement uncertainty may dominate the angular loss variations of the connectors.

Fiber loss (Table II) consists of the loss incurred in the 640 M glass fiber, two array splices, and the two half-connectors on the distribution frame. The average loss of the seven fibers measured is 5.96 ± 0.40 dB. Subtracting from it a mean loss of 0.55 dB for each array splice and 3.5 dB for the average fiber loss, the average full jumper loss would be 1.36 dB. Considering the small size of fiber sample measured, this crudely defined connector loss with mixed sockets lies within the bounds of the previously established loss of 1.75 dB with hydroformed socket and 1.22 dB with machined socket. This was also the first time the end-to-end absolute loss of a fiber in the field environment was accurately determined.

The loss measurement method described and tested can be applied to several areas of interest:

(*i*) To evaluate the comparative merits of optical jumpers with various types of connectors or sockets.

(*ii*) To perform precision loss measurement of optical fibers in a field environment.

(*iii*) To provide a means to estimate the aging and environmental effects on the optical fibers, jumpers, splices, and connectors.

(*iv*) When the absolute loss of a large number of jumpers or fibers needs to be determined, we may use this method to determine the absolute loss of a few elements. Then one can quickly switch over to simpler means such as using a variable attenuator, a transmission loss set, or an optical power meter to determine the relative loss between these elements and the rest.

The method, though tedious, is capable of making a simultaneous loss determination of six elements. Twenty-seven jumpers and fibers have been successfully measured. Some of them were measured many times to check consistency. The agreement among various measurements is generally good. Three elements show a definite increase of loss after hundreds of times of handling, indicating a possible permanent dirt penetration or surface damage on the connectors. A clear understanding and a substantial reduction, if not complete elimination, of such phenomena is needed before the real system usage.

It is also shown convincingly that connectors with machined sockets perform better than those with hydroformed sockets. However, even with machined sockets, the jumper loss variation of 0.5 to 1.9 dB is larger than desirable, particularly from the viewpoint of setting engineering rules for fiberguide systems.

VII. EVOLUTION OF THE SINGLE-FIBER CONNECTOR AFTER THE ATLANTA EXPERIMENT

The single-fiber connector obviously was in the initial development phase at the beginning of the Atlanta Experiment. Nevertheless, valuable information could be gained by the experiment itself, which led to a number of significant improvements. A molded biconic sleeve was developed by W. C. Young,[4] and the performance of the connector improved significantly. The mechanical tolerances of molded sleeves can be held more tightly than those of machined sleeves. The eccentricity error is less than 2 μm and, because of tighter control of the length parameter, the nominal end separation between fibers could be reduced to 25 μm.

A second significant change lies in the preparation of the fiber ends. The protruding fiber stub, as in Fig. 5, was found to be much too vulnerable to impact, and a fiber end flush with the plug was desirable. The breaking method of Gloge et al.[2] does not lend itself to produce flush ends. It was found that breaking a fiber closer to the plug body would result in break faces that were no longer perpendicular to the fiber axis, but showed progressively larger break angles (up to 60 degrees, when the fibers were scored somewhat inside the epoxy). We have therefore adopted a quick lapping and polishing process, which produces good, square fiber ends flush with the plug body.

More recently reported results[4] of 0.4-dB transmission loss for LED excitation and index match were a consequence of these implemented changes plus the improved characteristics of the optical fiber itself. The results were obtained by measuring the average transmission loss of 50

jumper cables against a standard connector as the final step in their inspection. Perhaps more indicative of the present state of the fiber and connector technology is the result obtained in January 1978. Twenty randomly selected jumper cables of the newest variety were connected in series at random connector orientations and excited with an LED. The measured connector losses had a mean of 0.54 dB and a standard deviation of 0.3 dB without index match between the fiber ends.

VIII. CONCLUSION

The Atlanta Fiber System Experiment was the testing ground for the first generation of fiber-optic interconnection hardware and has yielded valuable information for subsequent system experiments. An attempt was made to meet the challenge with a mass-produced single-fiber connector. While the technology was not completely developed at the time of the experiment, satisfactory results were nevertheless obtained.

IX. ACKNOWLEDGMENTS

We would like to thank our colleagues T. C. Chu, L. Curtis, A. R. McCormick, L. Maggi, C. R. Sandahl, and W. C. Young for their invaluable contributions to this success. The assistance by L. Wilson in loss data collection is also appreciated.

APPENDIX

Matrix Formulation of Maximum Likelihood Loss Measurement

Let x_i, $i = 1,5$ represent the loss of three jumpers and two fibers respectively and x_6 the baseline level of the set. The measured power level and its uncertainty are represented by y_i and σ_i, $i = 1,10$. The measurement sequence follows the order outlined in Fig. 14. Elements of x_i and y_i can be related in a matrix equation

$$Ax = y, \tag{1}$$

where x and y are column matrices. A is a 6-by-10 configuration matrix whose element is either 1 or 0, depending on whether the particular loss

element in question is present in the link. In the measurement

$$A = \begin{array}{c} \text{Loss element} \\ \\ \end{array} \begin{array}{cccccc} \text{J1} & \text{J2} & \text{J3} & \text{F1} & \text{F2} & \text{Set} \\ \end{array}$$

$$A = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{vmatrix} \qquad (2)$$

The sequence of measurements is chosen to minimize the number of changes of loss elements from one setup to another.

Since there are more equations than unknowns in the matrix equation (1), an exact or unique solution for x does not exist. However, a best-fit can be found by using a multiparameter least-square-fit technique[5] that is widely used by experimental physicists.

If we consider the measured results y_i to be Gaussian, then the likelihood function is proportional to

$$L(x_1,x_2,\cdots,x_n) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(y_i-\xi_i)^2}{2\sigma_i^2}\right], \qquad (3)$$

where $N = 10$ is the number of measurements and $n = 6$ is the number of unknowns. $\xi_i = \xi_i\,(x_1,x_2,\cdots x_n)$ is the "theoretical" value or the best fit value of y_i.

To maximize the likelihood L is equivalent to minimizing the exponent

$$\sum_{i=1}^{N} \frac{(y_i - \xi_i)^2}{2\sigma_i^2},$$

which leads to the condition of least-square fit, namely,

$$\sum_{i=1}^{N} \left(\frac{y_i - \xi_i}{\sigma_i^2}\right) \frac{\partial \xi_i}{\partial x_m} = 0, \quad m = 1,2,\cdots,n. \qquad (4)$$

Since

$$\xi_i = \sum_{m=1}^{n} A_{im} x_m, \; i = 1,2,\cdots,N$$

from eq. (1), the maximum likelihood condition of eq. (4) becomes

$$\sum_{i=1}^{N} \frac{A_{im} y_i}{\sigma_i^2} = \sum_{i,l=1}^{N} \frac{A_{im} A_{il}}{\sigma_i^2} x_l. \tag{5}$$

In matrix form, eq. (5) becomes

$$Y = Mx, \tag{6}$$

where

$$Y_m = \sum_{i=1}^{N} \frac{A_{im} y_i}{\sigma_i^2}, \quad m = 1,2,\cdots,n \tag{7}$$

is a modified data vector and

$$M_{ml} = \sum_{i=1}^{N} \frac{A_{im} A_{il}}{\sigma_i^2} = M_{lm}, \quad m,l = 1,2,\cdots,n \tag{8}$$

is a symmetric square matrix called the measurement matrix. The solution of eq. (6) is

$$x = M^{-1} Y. \tag{9}$$

The standard error in x_m is

$$\Delta x_m = (M_{mm}^{-1})^{1/2}, \quad m = 1,2,\cdots,n. \tag{10}$$

Because of the above property, M^{-1} is referred to as the error matrix. From eqs. (9) and (10), it is clear that a symmetric matrix inversion routine of rank 6-by-6 constitutes the bulk of the data reduction. The expected measured power is defined as

$$\xi_i = \sum_{m=1}^{n} A_{im} x_m \quad i = 1,2,\cdots,N \tag{11}$$

and the expected measured uncertainty

$$\eta_i = \left[\sum_{m=1}^{n} A_{im} \Delta x_m^2 \right]^{1/2}, \quad i = 1,2,\cdots,N \tag{12}$$

Comparison of ξ_i, η_i with y_i, σ_i can give a qualitative feeling of the quality of the loss measurement and the least-square fit.

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